

Signature of the interaction between dark energy and dark matter in galaxy clusters

Elcio Abdalla^a, L. Raul Abramo^a, Laerte Sodré Jr.^b, Bin Wang^c

^a*Instituto de Física, Universidade de São Paulo, CP 66318, 05315-970, São Paulo, Brazil*

^b*Departamento de Astronomia, Instituto de Astronomia, Geofísica e Ciências Atmosféricas da Universidade de São Paulo
Rua do Matão 1226, Cidade Universitária, 05508-090, São Paulo, Brazil*

^c*Department of Physics, Fudan University, 200433 Shanghai, China*

Abstract

We investigate the influence of an interaction between dark energy and dark matter upon the dynamics of galaxy clusters. We obtain the general Layser-Irvine equation in the presence of interactions, and find how, in that case, the virial theorem stands corrected. Using optical, X-ray and weak lensing data from 33 relaxed galaxy clusters, we put constraints on the strength of the coupling between the dark sectors. Available data suggests that this coupling is small but positive, indicating that dark energy might be decaying into dark matter. Systematic effects between the several mass estimates, however, should be better known, before definitive conclusions on the magnitude and significance of this coupling could be established.

Key words:

PACS: 98.80.C9, 98.80.-k

1. Introduction

Cosmological accelerated expansion is by now a well-established observational fact [1, 2, 3], leading either to an asymptotically de Sitter cosmology, plagued with an astonishingly small cosmological constant, or else to a universe filled up to 80% with a strange dynamical component with negative pressure – dark energy [4].

If dark energy contributes a significant fraction of the content of the Universe, it is natural, in the framework of field theory, to consider its interactions with the remaining fields of the Standard Model and well-motivated extensions thereof. For lack of evidence to the contrary, interactions of dark energy or dark matter with baryonic matter and radiation must be either inexistent or negligible. Nevertheless, some level of interaction between dark energy and the dark matter sector, which is present in most extensions of the Standard Model, is still allowed by observations.

The possibility that dark energy and dark matter can interact has been studied in [5]–[13], among others. It has been shown that the coupling between a dark energy (or quintessence) field and the dark matter can provide a mechanism to alleviate the coincidence problem [5, 10]. A suitable choice of the coupling, motivated by holographic arguments, can also lead to the crossing of the phantom barrier which separates models with $w = p/\rho > -1$ from models with $w < -1$ [11] – see also

[13, 14]. In addition, it has been argued that an appropriate interaction between dark energy and dark matter can influence the perturbation dynamics and affect the lowest multipoles of the CMB spectrum, accounting for the observed suppression of the quadrupole [12, 15]. The strength of the coupling could be as large as the fine structure constant [12, 16]. Recently, it was shown that such an interaction could be inferred from the expansion history of the universe, as manifested in, e.g., the supernova data together with CMB and large-scale structure [17]. Nevertheless, the observational limits on the strength of such an interaction remain weak [18].

A complementary and fundamentally different way in which the coupling between dark energy and dark matter can be checked against the observations is through its impact on large-scale structure. If dark energy is not a cosmological constant, it must fluctuate in space and in time – and, in particular, if dark energy couples to dark matter, then it must surely be dynamical. If that is the case, dark energy affects not only the expansion rate, but the process of structure formation as well, through density fluctuations, both in the linear [10], [19]–[22] and the non-linear [23, 24] regimes. The growth of dark matter perturbations can in fact be enhanced due to the coupling between these two components [12, 13, 25].

Recently, it was suggested that the dynamical equilibrium of collapsed structures would be affected by the coupling of dark energy to dark matter, in a way that could be observed in the galaxy cluster Abell A586 [26]. The basic idea is that the virial theorem is distorted by the non-conservation of mass caused by the coupling.

In this paper we show precisely how the Layser-Irvine

Email addresses: Email: eabdalla@fma.if.usp.br (Elcio Abdalla), Email: abramo@fma.if.usp.br (L. Raul Abramo), Email: laerte@astro.iag.usp.br (Laerte Sodré Jr.), Email: wangb@fudan.edu.cn (Bin Wang)

equation, which describes the flow to virialization [27], is changed by the presence of the coupling, in such a way that the final state of equilibrium violates the usual virial condition, $2K + U = 0$, where K and U are respectively the kinetic and the potential energies of the matter constituents in an isolated system. We show that this violation leads to a systematic bias in the estimation of masses of clusters if the usual virial conditions are employed. Although it is still possible that systematic errors from observations smear the results, the fact that some shift in the mean value of the coupling for two independent sets of observations (compared to the third set) may signalize some new physics.

Even though the uncertainties associated with any individual galaxy cluster are very large, by comparing the naive virial masses of a large sample of clusters with their masses estimated by X-ray and by weak lensing data, we may be able to constrain such a bias and to impose tighter limits on the strength of the coupling than has been achieved before.

2. PHENOMENOLOGY OF COUPLED DARK ENERGY AND DARK MATTER MODELS

Quite generically, at the level of the cosmological background an interaction between dark matter and dark energy manifests itself as a source term in the continuity equations of both fluids:

$$\begin{aligned}\dot{\rho}_{dm} + 3H\rho_{dm} &= \psi \\ \dot{\rho}_{de} + 3H\rho_{de}(1 + w_{de}) &= -\psi,\end{aligned}\quad (1)$$

where a dot denotes time derivative, H is the expansion rate, ρ_{dm} and ρ_{de} are respectively the energy densities of dark matter and dark energy, and w_{dm} and w_{de} are their equation of state parameter. Notice that the continuity equation still holds for the total energy density $\rho_{Tot} = \rho_{dm} + \rho_{de}$.

Phenomenologically, one can describe the interaction between the two fluids as an exchange of energy at a rate proportional to the total energy density [8, 11]:

$$\psi = \zeta H \rho_{Tot} . \quad (2)$$

We are interested in collapsed structures – places where the local, inhomogeneous density σ is far from the average, homogeneous density ρ . In that case the continuity equation for dark matter reads:

$$\dot{\sigma}_{dm} + 3H\sigma_{dm} + \vec{\nabla} \cdot (\sigma_{dm} \vec{v}_{dm}) = \zeta H (\sigma_{dm} + \sigma_{de}) , \quad (3)$$

where \vec{v}_{dm} is the peculiar velocity of dark matter particles.

In this work we will consider the local density of dark energy to be proportional to the local density of dark matter, $\sigma_{de} = b_{em}\sigma_{dm}$. If for a given model the dark energy component is very homogeneous, $b_{em} \approx 0$. We do not consider the case where b_{em} depends on the size and mass

of the collapsed structure – although this should probably happen in realistic models of structure formation with dark energy [24]. Hence, the continuity equation with dark matter coupled to dark energy reads:

$$\dot{\sigma}_{dm} + 3H\sigma_{dm} + \vec{\nabla} \cdot (\sigma_{dm} \vec{v}_{dm}) = \bar{\zeta} H \sigma_{dm} , \quad (4)$$

where $\bar{\zeta} = \zeta(1 + b_{em})$ is the effective coupling in a virialized structure. Notice that different dark energy models predict different levels of dark energy perturbations [24, 25], hence any constraints we derive from observations of collapsed structures will be in some sense degenerate with the perturbative properties of the dark energy sector.

3. LAYZER-IRVINE EQUATION IN THE PRESENCE OF COUPLING

We will use Newtonian mechanics to derive equilibrium conditions for a collapsed structure in an expanding Universe. The acceleration due to the gravitational force is given by:

$$(a \vec{v}_{dm})' = -a \vec{\nabla} \varphi , \quad (5)$$

where a is the scale factor and φ is the (Newtonian) gravitational potential. Multiplying both sides of this equation by $\sigma_{dm} a \vec{v}_{dm}$, integrating over the volume and using the continuity Eq. (4), we get that the left-hand side becomes:

$$(a^2 K_{dm})' = a^2 \bar{\zeta} H K_{dm} , \quad (6)$$

where the kinetic energy of dark matter is given by:

$$K_{dm} = \frac{1}{2} \int \vec{v}_{dm}^2 \sigma_{dm} dV . \quad (7)$$

The right-hand side of the equation, on the other hand, becomes:

$$(1 + b_{em}) \left[-a^2 (\dot{U}_{dm} + H U_{dm}) + 2\bar{\zeta} H a^2 U_{dm} \right] , \quad (8)$$

where we have used the Poisson equation, the fact that $\sigma_{Tot} = (1 + b_{em})\sigma_{dm}$, and the definition of the potential energy of a distribution of dark matter particles:

$$U_{dm} = -\frac{1}{2} G \int \int \frac{\sigma_{dm}(x) \sigma_{dm}(x')}{|x - x'|} dV dV' . \quad (9)$$

The identity between Eqs. (6) and (8) is the generalization of the Layzer-Irvine equation [27] describing how a collapsing system reaches a state of dynamical equilibrium in an expanding universe. One can see that the presence of the coupling between dark energy and dark matter changes both the time required by the system to reach equilibrium, and the equilibrium configuration itself. For a system in equilibrium ($\dot{K}_{dm} = \dot{U}_{dm} = 0$) we get the condition:

$$(2 - \bar{\zeta}) K_{dm} + (1 + b_{em})(1 - 2\bar{\zeta}) U_{dm} = 0 . \quad (10)$$

Taking $\bar{\zeta} = b_{em} = 0$ we recover the usual virial condition.

4. MASS ESTIMATION AND LIMITS ON THE COUPLING

Galaxy clusters are the largest virialized structures in the Universe, and their mass content is supposed to be representative of the universe as a whole – see, e.g., [28]. They are composed of hundreds of galaxies, with the largest fraction of their baryonic mass in the form of hot, X-ray emitting gas – not stars. Clusters are widely believed to be totally dominated by dark matter [29], and are conspicuous: existing surveys have already detected many thousands of clusters, and upcoming surveys will map much more.

Cluster masses can be estimated in a variety of ways. Weak lensing methods use the distortion in the pattern of images behind the cluster (which acts as a lens) to compute the projected gravitational potential due to that cluster. Knowing the distances to the cluster and to the background images, one can derive the mass that causes that potential. An independent mass estimation can be obtained from X-ray observations if we assume that the ionized gas is in hydrostatic equilibrium. In this case the cluster mass can be determined by the condition that the gravitational attraction is supported by the gas pressure. Finally, we can measure radial velocities and the projected distribution of cluster galaxies and, by assuming that clusters are virialized, one can infer their masses using the fact that $U \propto \sigma^2$ but $K \propto \sigma$, hence $U/K \propto M$.

However, Eq. [10] tells us that when there is coupling between dark matter and dark energy, the equilibrium condition depends on the coupling as:

$$(1 + b_{em}) \frac{U_{dm}}{K_{dm}} = -2 \frac{1 - \bar{\zeta}/2}{1 - 2\bar{\zeta}}. \quad (11)$$

Hence, the mass that is estimated under the assumption that $\bar{\zeta} = 0$ is biased with the respect to the actual mass by a factor of $(1 - \bar{\zeta}/2)/(1 - 2\bar{\zeta})$.

One can compare directly the mass obtained through the virial theorem with that determined by other methods. Notice that the total mass of a cluster is, within our approximations, the integral of $\sigma_{Tot} = (1 + b_{em})\sigma_{dm}$ over the volume; hence, the mass estimated through X-rays and weak lensing is related to the virial mass as $M = (1 + b_{em})M_{dm}$.

Therefore, comparing the masses estimated through the **conventional** virial hypothesis with the masses estimated either by weak lensing or by X-ray data, we get:

$$M_X = M_{WL} = M_{vir} \times \frac{1 - \bar{\zeta}/2}{1 - 2\bar{\zeta}} \quad (12)$$

Hence, there are three tests one can make with these datasets:

$$f_1 = M_X/M_{vir}, \quad (13)$$

$$f_2 = M_{WL}/M_{vir}, \quad (14)$$

$$f_3 = M_X/M_{WL}. \quad (15)$$

If our interaction model is right, the first two tests, f_1 and f_2 , should agree with each other and put similar limits on the effective coupling parameter $\bar{\zeta}$, while the third test, f_3 , should only be a check of our method, and its value should be equal to one unless there are unknown systematics affecting our mass estimates. Notice that either a violation of the equivalence principle for dark matter or a self-interaction of dark matter with itself, such as suggested by [30], could also be tested by comparing the different mass estimates – see also [26].

To what extent the data currently available on cluster masses allows us to constrain the coupling parameter? To compare masses obtained with the different methods, we have assumed that the mass profile of the clusters is described by a Singular Isothermal Sphere (SIS). The main reasons are that weak-lensing mass estimations need to adopt a parametric model for the mass distribution in order to avoid the so-called mass-sheet degeneracy [31]; this model is largely adopted in weak lensing studies, and most weak-lensing and X-ray mass estimations are possible only for radii significantly smaller than the virial. The main advantage of this model is that it has a single parameter – the velocity dispersion along the line-of sight σ_v – which can be easily determined: directly from the observed radial velocities in the virial estimation; from the X-ray temperature, $[\sigma_X^2 = kT_X/(\mu m_H)]$, where $\mu=0.61$ is the mean molecular weight]; and from the fitting of the shear field in the case of weak-lensing. Since in this model the mass inside a given (projected) radius R is $M(< R) = \pi\sigma^2 R/G$, to compare the masses obtained by each method we need only to compare the velocity dispersions.

For this exercise, we have analyzed data from galaxy clusters studied in [31, 32, 33]. Our sample has 33 clusters and was selected due to the homogeneity in the analysis procedure and avoiding clusters with evidence of dynamical activity, like substructures. Ref. [31] presents a weak-lensing analysis of 24 galaxy clusters also observed in X-rays, verifying that clusters with a hot intergalactic medium ($T_X > 8$ keV) are very active. For our analysis, we selected from this paper 14 clusters (from the 15 clusters with X-ray temperatures lower than 8 keV, Abell 1651 also has evidence of significant substructure [34]). The cluster A586, discussed in [32], also seems to be in equilibrium. The remaining 18 clusters of our sample comes from [33].

We have used this dataset to test the theory that the usual virial mass is biased by a factor $(1 - 2\bar{\zeta})/(1 - \bar{\zeta}/2)$ when compared to other mass estimates. Although the three datasets have asymmetric errors, we have assumed that the likelihood function associated with the three tests is symmetric, with width $\sigma = \sqrt{\sigma_+ \sigma_-}$. With these assumptions, the likelihood function of test i is:

$$\mathcal{L}_i \propto \prod_{n=1}^{N_i} \exp \left\{ -\frac{1}{2\sigma_i^2(n)} \left[\frac{1 - 2\bar{\zeta}_i}{1 - \bar{\zeta}_i/2} f_i(n) \right]^2 \right\}, \quad (16)$$

where the product runs over the data for each galaxy cluster n . If our model is correct we should get $\bar{\zeta}_1 = \bar{\zeta}_2 = \bar{\zeta}$,

and $\bar{\zeta}_3 = 0$.

In the three panels of Fig. 1 we show the probability distribution functions for f_1 , f_2 and f_3 , all computed for the top-hat prior $-0.2 < \bar{\zeta}_i < 0.2$, which is more than sufficient to include the $3\text{-}\sigma$ limits $-0.12 < \bar{\zeta}_i < 0.06$ found by [18]. The shadowed regions in the top and middle panels mark the 67% and 95% Confidence Level (C.L.) limits for $\bar{\zeta}_1$ and $\bar{\zeta}_2$.

For both the f_1 and f_2 tests we get a best-fit value of $\bar{\zeta} \sim 0.03\text{--}0.04$, while for the check f_3 we indeed get that $\bar{\zeta}_3$ is consistent with zero with a high statistical significance. Our 95% C.L. limits are $0.0 \lesssim \bar{\zeta} \lesssim 0.06$ for the test f_1 and $0.0 \lesssim \bar{\zeta} \lesssim 0.09$ for f_2 . This compares favourably with the 95% C.L. constraint $-0.095 \lesssim \bar{\zeta} \lesssim 0.035$ obtained by [18].

For f_1 and f_2 we obtain that the null hypothesis ($\bar{\zeta} = 0$) is marginally consistent with the data, at the edge of the 95% C.L. region. The statistical improvement between the null hypothesis and the best-fit model with coupling is weak, though: we get a $\Delta\chi^2/\text{d.o.f.} \sim 0.2$ for both the f_1 and f_2 tests.

We have also computed the Bayesian Information Criterion (BIC) [35] to weigh if, and by how much, a coupling is necessary. For the test f_1 we get $\Delta_{\text{BIC}}^1 \approx -0.5$, while for the test f_2 we get $\Delta_{\text{BIC}}^2 \approx -2.1$. We can also estimate the level of systematic uncertainties that would turn the BIC against our model (i.e., when $\Delta_{\text{BIC}} = 0$): an enhancement of 20% of all uncertainties would make $\Delta_{\text{BIC}}^1 \approx 0$, but still $\Delta_{\text{BIC}}^2 \approx -1.2$; in order to make $\Delta_{\text{BIC}}^2 \approx 0$ it would take an enhancement of 70% of the uncertainties.

The reliability of these constraints, however, are disputable, due to possible systematic effects in the mass determinations. For example, a virial mass estimate is affected by the assumptions about the galaxy orbits, cluster morphology, mass distribution, identification of interlopers, etc. [33, 36, 37], and the robustness of our simple SIS model does not mean that it is insensitive to (unknown) systematics. We can have a hint on the impact of these effects for the constraints on the effective coupling constant by increasing its possible range of variation. For this exercise we have just redone the analysis assuming that the actual errors are twice the internal errors. In this case the original results ($\bar{\zeta}_1 = 0.029 \pm 0.015$ and $\bar{\zeta}_2 = 0.044 \pm 0.027$ at the 68% C.L.) change to $\bar{\zeta}_1 = 0.029 \pm 0.030$ and $\bar{\zeta}_2 = 0.044 \pm 0.037$, i.e., the most probable value of $\bar{\zeta}_{1,2}$ does not change (as expected in this case), but the error in the estimates almost doubles, reducing to about one sigma the level of detection of a non-zero coupling constant. Moreover, since we can only constrain the effective coupling parameter $\bar{\zeta} = (1 + b_{em})\zeta$, the bias between dark matter and dark energy in virialized structures could enhance (if $b_{em} > 0$) or suppress ($b_{em} < 0$) our ability to constrain the true coupling ζ .

These results show that the reliability of a detection of the effective coupling parameter requires very good knowledge of possible systematic errors. Nevertheless, it also shows that if in the future we can produce a sample with reliable mass estimates and controlled systematics, we will

indeed be able to constrain $\bar{\zeta}_{1,2}$ and verify whether this hint of a coupling between dark matter and dark energy found with current data is confirmed.

5. CONCLUSIONS

We have estimated the effective coupling between dark energy and dark matter through the internal dynamics of galaxy clusters. In the presence of coupling, the flow of mass and energy between the components changes the virial condition in a way that can be tested by comparing different estimators for the mass of clusters. We searched for this signature in 33 galaxy clusters for which reliable X-ray, weak lensing and optical data were available.

Our results indicate a weak preference for a small but positive effective coupling constant $\bar{\zeta}$ – in line with predictions made by some of us [11, 12]. Since the statistical significance is still low ($\Delta\chi^2/\text{d.o.f.} \sim 0.2$), it is paramount that more clusters (with homogeneous mass determinations and good control of systematics) be tested. If a significant indication of such coupling is still found, this would open a tantalizing new window on the nature of the dark sector.

Acknowledgments

This work has been supported by FAPESP and CNPq of Brazil, by NNSF of China, Shanghai Education Commission, and Shanghai Science and Technology Commission.

References

- [1] A. G. Riess et al., *Astron. J.* **116**: 1009 (1998); S. Perlmutter et al., *Astrophys. J.* **517**: 565 (1999); P. de Bernardis et al., *Nature* **404**: 955 (2000); R. Knop et al., *Astrophys. J.* **598**: 102 (2003); A. G. Riess et al., *Astrophys. J.* **607**: 665 (2004); P. Astier et al., *Astron. Astrophys.* **447**: 31 (2006); W. M. Wood-Vasey et al., *Astrophys. J.* **666**: 694 (2007), astro-ph/0701041.
- [2] WMAP, D. N. Spergel et al., *Astrophys. J. Suppl.* **170**: 377 (2007).
- [3] SDSS, D. J. Eisenstein et al., *Astrophys. J.* **633**: 560 (2005).
- [4] T. Padmanabhan, *Phys. Rept.* **380**: 235 (2003), hep-th/0212290; P. J. E. Peebles and B. Ratra, *Rev. Mod. Phys.* **75**: 559 (2003), astro-ph/0207347; V. Sahni, *Lect. Notes Phys.* **653**: 141 (2004), astro-ph/0403324.
- [5] L. Amendola, *Phys. Rev.* **D62**: 043511 (2000); Luca Amendola and Claudia Quercellini, *Phys. Rev.* **D68**: 023514 (2003); Luca Amendola, Shinji Tsujikawa and M. Sami, *Phys. Lett.* **B632**: 155 (2006).
- [6] M. Pietroni, *Phys. Rev.* **D67**: 103523 (2003); D. Comelli, M. Pietroni and A. Riotto, *Phys. Lett.* **B571**: 115 (2003).
- [7] G. Farrar and P. J. E. Peebles, *Astrophys. J.* **604**: 1 (2004).
- [8] D. Pavon, W. Zimdahl, *Phys. Lett.* **B628**: 206 (2005), gr-qc/0505020.
- [9] S. Campo, R. Herrera, G. Olivares and D. Pavon, *Phys. Rev.* **D74**: 023501 (2006); S. Campo, R. Herrera and D. Pavon, *Phys. Rev.* **D71**: 123529 (2005); G. Olivares, F. Atrio-Barandela and D. Pavon, *Phys. Rev.* **D71**: 063523 (2005).
- [10] G. Olivares, F. Atrio-Barandela, D. Pavon, *Phys. Rev.* **D74**: 043521 (2006).
- [11] B. Wang, Y. G. Gong and E. Abdalla, *Phys. Lett.* **B624**: 141 (2005); B. Wang, C. Y. Lin and E. Abdalla, *Phys. Lett.* **B637**: 357 (2006).

- [12] B. Wang, J. Zang, C.Y. Lin, E. Abdalla and S. Micheletti, *Nucl. Phys.* **B778**: 69 (2007).
- [13] S. Das, P. S. Corasaniti and J. Khoury, *Phys. Rev.* **D73**: 083509 (2006).
- [14] G. Huey and B. Wandelt, *Phys. Rev.* **D74**: 023519 (2006).
- [15] W. Zimdahl, *Int. J. Mod. Phys.* **D14**: 2319 (2005), gr-qc/0505056.
- [16] E. Abdalla and B. Wang *Phys. Lett.* **B651**: 89 (2007).
- [17] C. Feng, B. Wang, Yungui Gong and Ru-Keng Su, *JCAP* **0709**: 005 (2007), arXiv:0706.4033.
- [18] Z. K. Guo, N. Ohta and S. Tsujikawa, *Phys. Rev.* **D76**: 023508 (2007), astro-ph/0702015.
- [19] P. Ferreira and M. Joyce, *Phys. Rev.* **D58**: 023503 (1998).
- [20] P. Viana and A. Liddle, *Phys. Rev.* **D57**: 674 (1998).
- [21] L. R. Abramo and F. Finelli, *Phys. Rev.* **D64**: 083513 (2001).
- [22] R. Bean, *Phys. Rev.* **D64**: 123516 (2001).
- [23] N. Nunes and D. Mota, *Mon. Not. Roy. Astron. Soc.* **368**: 751 (2006).
- [24] L. R. Abramo, R. C. Batista, L. Liberato and R. Rosenfeld, *JCAP* **0711**: 012 (2007), arXiv: 0707.2882 [astro-ph]
- [25] M. Manera and D. Mota, *Mon. Not. Roy. Astron. Soc.* **371**: 1373 (2006).
- [26] O. Bertolami, F. Gil Pedro and M. Le Delliou *Phys. Lett.* **B654**: 165 (2007), astro-ph/0703462; idem, arXiv:0705.3118 [astro-ph]; idem arXiv:0801.0201 [astro-ph]
- [27] P. J. E. Peebles, “Physical Cosmology” (Princeton U. Press, Princeton, 1993).
- [28] S. Eddington, *Mon. Not. Roy. Astron. Soc.* **344**: L13 (2003).
- [29] N. Bahcall, in “Formation of Structure in the Universe”, ed. A. Dekel & J. P. Ostriker (Cambridge University Press, 1999).
- [30] M. Kesden and M. Kamionkowski, *Phys. Rev. Lett.* **97**: 131303 (2006); idem, *Phys. Rev. D* **74**: 083007 (2006).
- [31] E. S. Cypriano, L. Sodré, J.-P. Kneib and L. E. Campusano, *Astrophys. J.* **613**: 95 (2004).
- [32] E. S. Cypriano, G. B. Lima Neto, L. Sodré, J.-P. Kneib and L. E. Campusano, *Astrophys. J.* **630**: 38 (2005).
- [33] H. Hoekstra, *Mon. Not. Roy. Astron. Soc.* **379**: 317 (2007).
- [34] P. Flin and J. Krywult, *Astron. Astrophys.* **450**: 9 (2006).
- [35] A. Liddle, *Mon. Not. Roy. Astron. Soc. Lett.* **377**: L74 (2007).
- [36] L. Sodré, H. V. Capelato, J. E. Steiner, A. Mazure, *Astronom. J.* **97**: 1279 (1989).
- [37] E. De Filippis, M. Sereno, M. W. Bautz, G. Longo, *Astrophys. J.* **625**: 108 (2005).

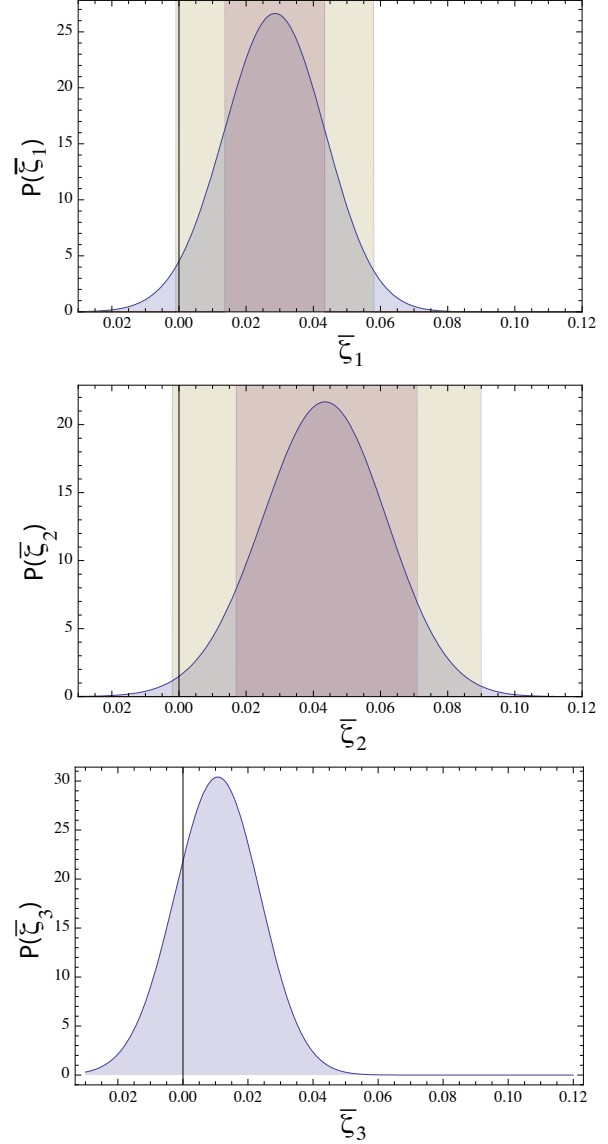


Figure 1: Normalized probability distribution functions for the tests f_1 (top panel), f_2 (middle panel) and f_3 (bottom panel.) The shaded regions on the top and middle panels indicate the 67% and 95% C.L. limits.